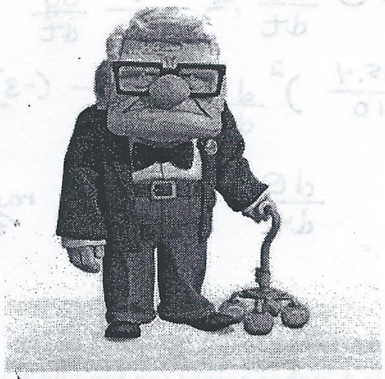
Name: Key - 40pts

Calculus - Related Rates Test

Solve each related rate problem. Show your equations and a diagram if needed. (10 pts ea)

1. Carl, from the movie UP, is blowing up another spherical balloon at the rate of 5 cubic inches per min. Find the rate of change of the radius when the radius equals 2.5 inches.



$\frac{dr}{dt} = ?$ when $r = 2.5$

$\frac{dV}{dt} = 5$

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$5 = 4\pi(2.5)^2 \frac{dr}{dt}$

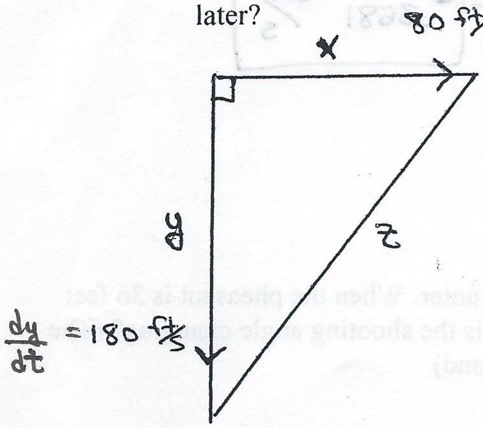
$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$ 4π

$\frac{5}{4\pi(2.5)^2} = \frac{dr}{dt} = \boxed{.0634 \text{ in/min}}$

25π

2. A cropduster is flying south at 180 ft/s. A car is traveling east on Highway 30 at 80 ft/s. The plane flies directly over the car at 3:00pm. How fast is the ground distance between the the two changing 10 seconds later?

$80 \text{ ft/s} = \frac{dx}{dt}$



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$800(80) + 1800(180) = 1970 \frac{dz}{dt}$

$\boxed{\frac{dz}{dt} = 197 \text{ ft/s}}$

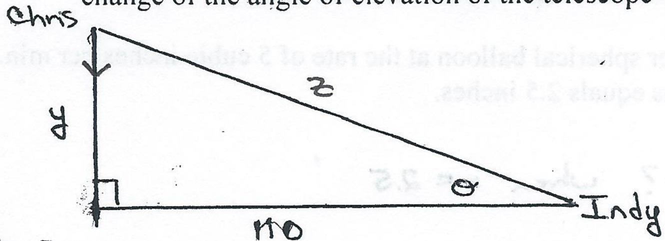
@ 10 sec

$x = 800$

$y = 1800$

$z = \sqrt{1800^2 + 800^2} = 1970$

3. Indy is hiding in a bush outside of the Gateway Hotel in Ames. He is looking for Chris with a telescope. He happens to see him as he climbs into the glass elevator on the fifth floor, 50 feet above Indy. If the elevator descends at 3 feet per second, and Indy is 110 feet from the base of the elevator shaft, what is the rate of change of the angle of elevation of the telescope when the elevator is 35 feet off the ground?



$\frac{dy}{dt} = -3$

$\frac{d\theta}{dt} = ?$ when $y = 35$

@ $y = 35$

$z = \sqrt{35^2 + 110^2} = 115.4$

$\tan\theta = \frac{y}{110}$

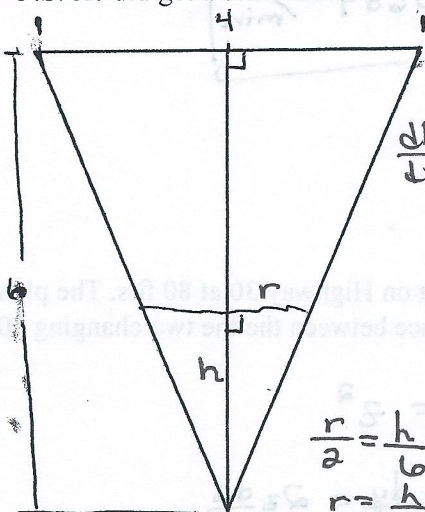
$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{110}\frac{dy}{dt}$

$\left(\frac{115.4}{110}\right)^2 \frac{d\theta}{dt} = \frac{1}{110}(-3)$

$\frac{d\theta}{dt} = -.0248$ rad/sec

4. Nick is eating a snow cone. The paper cone he is eating it from is 4 inches across the top and 6 inches tall. If he is eating the snow cone at a rate of 0.5 cubic inches/s, how fast is the slushie level dropping when it is 2 inches deep? (Assume it was only filled to the top of the cone)
P.S. He did get a brain freeze!

$\frac{dV}{dt} = -.5$



$h = 2$

$\frac{dh}{dt} = ?$

$\frac{r}{2} = \frac{h}{6}$

$r = \frac{h}{3}$

$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h = \frac{\pi}{27}h^3$

$\frac{dV}{dt} = \frac{\pi}{9}\cdot h^2 \frac{dh}{dt}$

$-.5 = \frac{\pi}{9}(2)^2 \frac{dh}{dt}$

$\frac{-.5 \cdot 9}{4\pi} = \frac{dh}{dt} = \boxed{-.3581 \text{ in/s}}$

Calc. 2.6 Test　　　　　　　　　　　　Name Key　　　40 pts

*** Open Notes, each problem is worth 10 points ***

1. As a spherical balloon is being blown up, the volume is increasing at the rate of 4 cubic inches per second. At what rate is the radius increasing when the radius is 1 inch?

$V = \frac{4\pi}{3} r^3$,　　Given: $\frac{dV}{dt} = 4$

Find: $\frac{dr}{dt} = ?$　$r = 1$

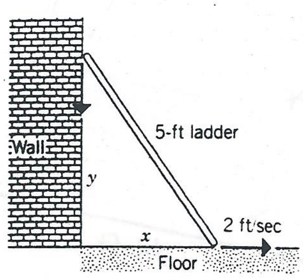
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$4 = 4\pi (1)^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{1}{\pi} = .3183 \text{ in/sec}$$

-1 for switching rates

2. A 5-ft ladder, leaning against a wall slips so that its base moves away from the wall at a rate of 2 ft/sec. How fast will the top of the ladder be moving down the wall when the base is 4 ft from the wall?



Equation

$x^2 + y^2 = 25$

Given: $\frac{dx}{dt} = 2$

Find: $\frac{dy}{dt} = ?$　$x = 4$

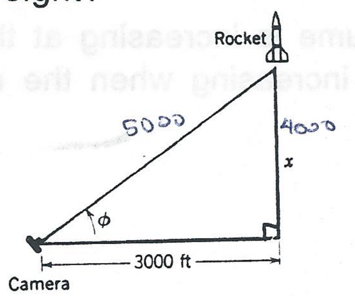
then $y = 3$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(4)(2) + 2(3) \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{8}{3} = -2.67 \text{ ft/sec}$$

3. If the rocket shown below is rising vertically at 880 ft/sec when it is 4000 ft up, how fast must the camera elevation angle change at that instant to keep the rocket in sight?



Equation

$\tan\phi = \dfrac{x}{3000}$

Given: $\dfrac{dx}{dt} = 880$ ft/sec

Find: $\dfrac{d\phi}{dt} = ?$ $x = 4000$

$\sec^2\phi \dfrac{d\phi}{dt} = \dfrac{1}{3000}\dfrac{dx}{dt}$

$\sec\phi = \dfrac{5}{3}$

$\left(\dfrac{5}{3}\right)^2 \dfrac{d\phi}{dt} = \dfrac{880}{3000}$

$\dfrac{d\phi}{dt} = .1056$ rad/sec

-2 wrong

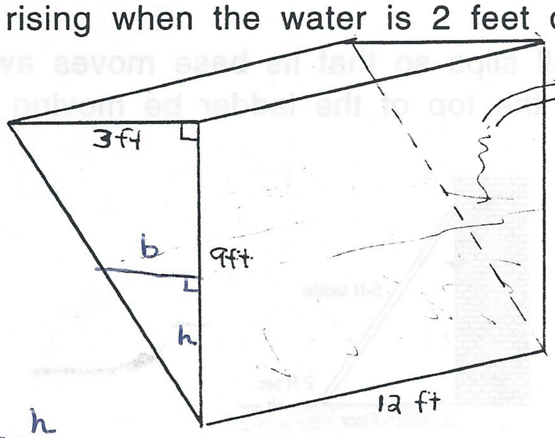
-3 no θ

-1 math!!

$\sec^2 = 2.76$

6/9

4. A trough is 12 feet long, 3 feet across the top, and 9 feet high (see figure). If water is being pumped into the trough at 1.5 cubic feet per minute, how fast is the water level rising when the water is 2 feet deep?



1.5 ft^3/min

Given

$\dfrac{dV}{dt} = 1.5$

Find

$\dfrac{dh}{dt} = ?$ $h = 2$

$\dfrac{b}{3} = \dfrac{h}{9}$

$b = \dfrac{h}{3}$

-3 wrong

no -4

-2

$V = Bh$

$V = \left(\frac{1}{2}bh\right)12$

$V = 6\left(\frac{h}{3}\right)h$

$V = 2h^2$

$\dfrac{dV}{dt} = 4h\dfrac{dh}{dt}$

$1.5 = 4(2)\dfrac{dh}{dt}$

$\dfrac{dh}{dt} = .1875$ ft/sec

Name: Key 40 pts

Calculus - 2.6 Related Rates Test

Solve each related rate problem. Show your equations and a diagram if needed. (10 pts ea)

1. The radius of a really huge, almost ginormous, pizza is increasing at the rate of 6mm per minute. Find the rate of change in the area when the radius is 36mm.

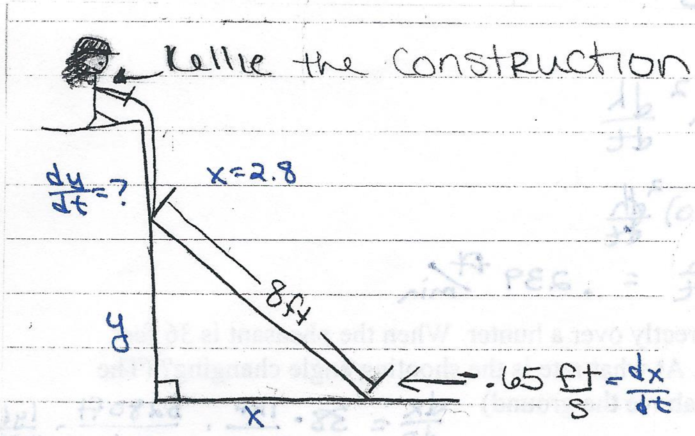
$A = \pi r^2$

$\frac{dr}{dt} = 6 \quad \frac{dA}{dt} = ? \quad r = 36$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(36)6 = 1357.2 \ \frac{mm^2}{min}$

2. Kellie is pulling an 8 foot board up the side of a building. She is pulling at a rate so that the end of the board slides along the ground at a rate of -.65 feet/second. How fast is the end of the plank moving up the wall when the bottom of the board is 2.8 feet from the wall?



$x^2 + y^2 = 8^2$

$2.8^2 + y^2 = 8^2$

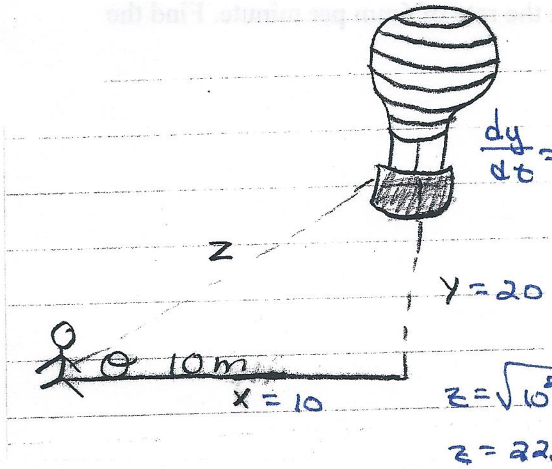
$y = 7.5$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2.8(-.65) + 7.5\frac{dy}{dt} = 0$

$\frac{dy}{dt} = .24 \ ft/sec$

3. Dorothy needs to catch a hot air balloon ride back to Kansas. Unfortunately, when she's 10m away, the balloon starts rising at the rate of 5 m/s. What is the rate of change of the angle of elevation of the hot air balloon as seen by Dorothy when the balloon is 20 meters off the ground?



$\frac{dy}{dt} = 5$

$\frac{d\theta}{dt} = ?$ $y = 20$

$z = \sqrt{10^2 + 20^2} = \sqrt{500}$

$z = 22.4$

~~$\sin\theta = \frac{y}{z}$~~

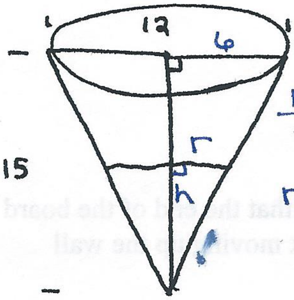
$\tan\theta = \frac{y}{10}$

$\sec^2\theta \, \frac{d\theta}{dt} = \frac{1}{10} \cdot \frac{dy}{dt}$

$\left(\frac{\sqrt{500}}{10}\right)^2 \cdot \frac{d\theta}{dt} = \frac{1}{10} \cdot 5$

$\frac{d\theta}{dt} = \frac{1}{10} = .1$

4. A conical tank is 12 feet across the top and 15 feet deep. If the water is flowing into the tank at a rate of 12 cubic feet/min, find the rate of change of the depth of the water when the water is 10 feet deep.



$\frac{dV}{dt} = 12$ $\frac{dh}{dt} = ?$, $h = 10$

$\frac{r}{6} = \frac{h}{15}$

$r = \frac{2h}{5}$

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \left(\frac{2h}{5}\right)^2 h$

$V = \frac{4\pi}{75} h^3$

$\frac{dV}{dt} = \frac{12\pi}{75} h^2 \frac{dh}{dt}$

$12 = \frac{12\pi}{75}(10)^2 \frac{dh}{dt}$

$\frac{75}{100\pi} = \frac{dh}{dt} = .239$ ft/min

Extra Credit (3 pts)

Name: ____________________________

Calculus - Related Rates Test

Solve each related rate problem. Show your equations and a diagram if needed. (10 pts ea)

1. A life size replica of Mr. D's head is being created. The volume of the model is increasing due to super ego problems at a rate of 6 cubic in/min. At what rate is the radius of Mr. D's head increasing when the radius is 3 inches. Assume Mr. D's head is in the shape of a sphere.

$\frac{dV}{dt} = 6$

$\frac{dr}{dt} = ?$ when $r = 3$

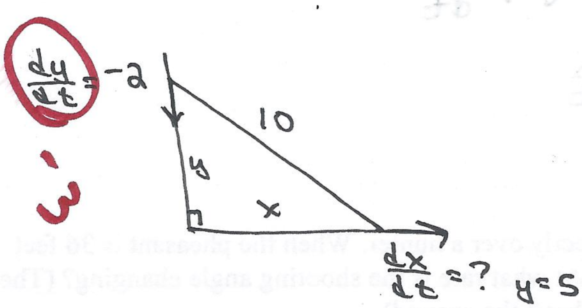
$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$6 = 4\pi (3)^2 \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = .053}$

2. A 10 ft grandfather clock had leaned against a wall for over 100 years. One day, an earthquake struck, and the clock began to slide down the wall at a rate of 2 ft/sec. How fast will the bottom of the clock move away from the wall when the top of the clock is 5 ft above the floor?



$x^2 + 5^2 = 10^2$

$x = \sqrt{75}$

$= 5\sqrt{3}$

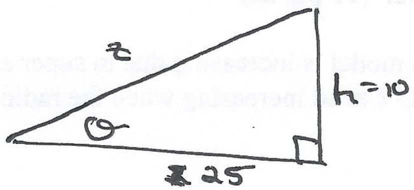
$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$(5\sqrt{3})\frac{dx}{dt} + (5)(-2)$

$\frac{dx}{dt} = \frac{+10}{5\sqrt{3}} = \frac{-2}{\sqrt{3}} = \boxed{+1.155}$

3. A UFO captures Jolyn and is taking her back to Mars. Jolyn's parents are standing 25m away. The UFO is rising at a rate of 15 m/s. What is the rate of change of the angle of elevation of the UFO from Barb and Joel, when the UFO is 10 m off the ground?



$$\tan\theta = \frac{h}{25}$$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{25}\frac{dh}{dt}$$

$$\left(\frac{26.93}{25}\right)^2 \frac{d\theta}{dt} = \frac{1}{25}(15)$$

$$\frac{dh}{dt} = 15$$

$$\frac{d\theta}{dt} = ?,\quad h = 10$$

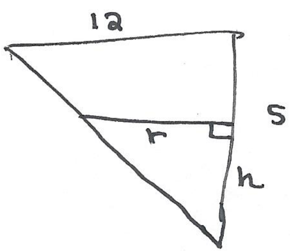
$$\sec\theta = \frac{\sqrt{725}}{25}$$

$$\frac{d\theta}{dt} = \frac{15}{26.93} = \cancel{.557}\ .517$$

$$z = \sqrt{100 + 625} = \sqrt{725}$$

$$= 26.93$$

4. A fire tank is 12 ft across the top, 4 ft wide and 5 feet high. If the water is being pumped into the tank at 133.69 cubic feet per minute, how fast is the water level rising when the water is 3 feet deep?



$$V = \tfrac{1}{2} rh \cdot 4$$

$$= \tfrac{1}{2}\left(\frac{12h}{5}\right)\cdot 4h$$

$$V = \frac{24}{5}h^2$$

$$\frac{dV}{dt} = \frac{48}{5}h\frac{dh}{dt}$$

$$\frac{r}{12} = \frac{h}{5}$$

$$133.69 = \frac{48}{5}(3)\frac{dh}{dt}$$

$$r = \frac{12h}{5}$$

$$4.64 = \frac{dh}{dt}$$

Name: Key 40 pts

Calculus - 2.6 Related Rates Test

Solve each related rate problem. Show your equations and a diagram if needed. (10 pts ea)

1. A magic wheel of cheese has a radius the is increasing at 10 feet/minute. Find the rate of change in the area when the radius is 50 ft.(This is one huge block of cheese!!!)

$\frac{dr}{dt} = 10 \quad \frac{dA}{dt} = ?, \ r = 50$

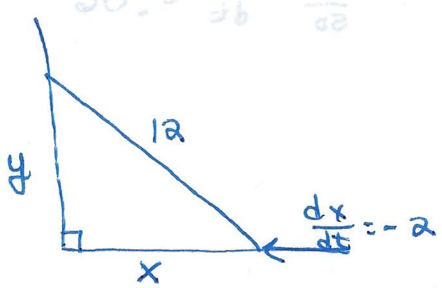
$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(50)(10)$

$= 1000\pi = 3.14(10)^3$

2. Andy the hippie is moving into a new building. He is dragging a 12 ft couch up the side of a building. He is pulling at a rate that the end of the couch slides along the ground at a rate of 2 feet/sec. How fast is the end of the couch moving up the wall when the bottom of the couch is 5 feet from the wall?



$\frac{dy}{dt} = ?, \ x = 5 \qquad 5^2 + y^2 = 12^2$

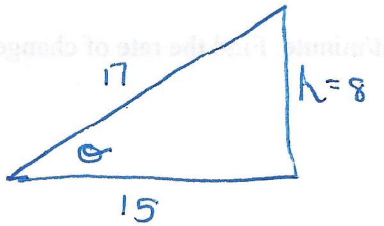
$x^2 + y^2 = 12^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$2(5)(-2) + 2(\sqrt{119})\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{10}{\sqrt{119}} = .92 \ ft/s$

3. An elevator is rising at a rate of 10 m/s. David is standing 15 m away from the elevator shaft when the elevator begins to rise? What is the rate of change of the angle of elevation of the elevator, when the elevator is 8 m off the ground?



$\frac{dh}{dt} = 10$

$\frac{d\theta}{dt} = ?,\ h = 8$

$\tan\theta = \frac{h}{15}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{15}\frac{dh}{dt}$

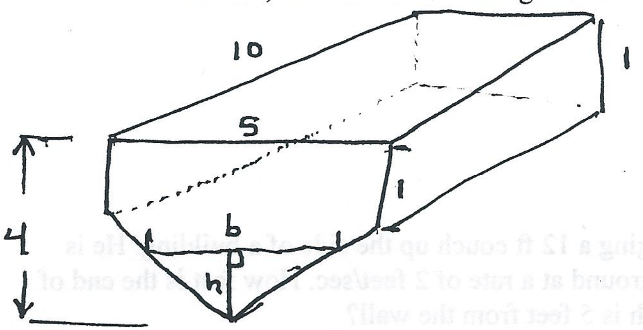
$\left(\frac{17}{15}\right)^2 \frac{d\theta}{dt} = \frac{1}{15}(10)$

$\sec\theta = \frac{17}{15}$

$\frac{d\theta}{dt} = \cancel{\frac{17^2 \cdot 10}{15}}\ \ \frac{15\cdot 10}{17^2}$

$= .519$

4. David Kim is filling up his hot tub because the CHS Volleyball team is coming over soon. The hot tub is 10 feet across, 5 feet wide and 4 feet deep (see diagram). If the water is flowing into the tank at a rate of 2 cubic feet/min, find the rate of change of the depth of the water when the water is 2 feet deep.



$\frac{dV}{dt} = 2$

$\frac{dh}{dt} = ?\ \ h = 2$

$V = \frac{1}{2}bh(10)$

$V = \frac{50}{6}h^2$

$\frac{dV}{dt} = \frac{100}{6}h\frac{dh}{dt}$

$2 = \frac{50}{3}(2)\frac{dh}{dt}$

$\frac{3}{50} = \frac{dh}{dt} = .06$

$\frac{b}{5} = \frac{h}{3}$

$b = \frac{5h}{3}$

Calc. 2.6 Test   Name___Key___

*** Open Notes, each problem is worth 10 points ***

1. As a spherical balloon is being blown up, the volume is increasing at the rate of 8 cubic inches per second. At what rate is the radius increasing when the radius is 2 inches?

$\frac{dV}{dt} = 8 \qquad \frac{dr}{dt} = ? \qquad r = 2$

$V = \frac{4\pi}{3} r^3$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

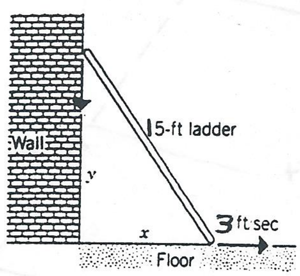
$8 = 4\pi (2)^2 \frac{dr}{dt}$

$\frac{1}{2\pi} = \frac{dr}{dt}$

$= .1592$ in/sec

-4 for switching rates

2. A 15-ft ladder, leaning against a wall slips so that its base moves away from the wall at a rate of 3 ft/sec. How fast will the top of the ladder be moving down the wall when the base is 4 ft from the wall?



$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = ? \qquad x = 4$

$x^2 + y^2 = 225$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$y^2 = 225 - 16$

$y = \sqrt{209} = 14.5$

$2(4)(3) + 2(\sqrt{209})\frac{dy}{dt} = 0$

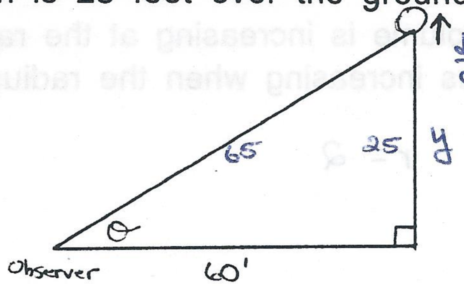
$\frac{dy}{dt} = \frac{-12}{\sqrt{209}} = -.83$ ft/sec

-3 for wrong 4

-1 for -

-3 for $\frac{dx}{dt}$ / $\frac{dy}{dt}$

3. A balloon rises at the rate of 8 feet per second from a point on the ground 60 feet from an observer. Find the rate of change of the angle of elevation when the balloon is 25 feet over the ground.



$\frac{d\theta}{dt} = ?$ $y = 25$

$\tan\theta = \frac{y}{60}$

$\frac{d\theta}{dt}\sec^2\theta = \frac{1}{60}\frac{dy}{dt}$

$60\left(\frac{13}{12}\right)^2\frac{d\theta}{dt} = \frac{dy}{dt}$

~~70.42 ft/sec = dy/dt~~

$\sec\theta = \frac{65}{60}$

$= \frac{13}{12}$

-2 wrong $\frac{d\theta}{dt}$

θ = 22.62°

(wrong thing)

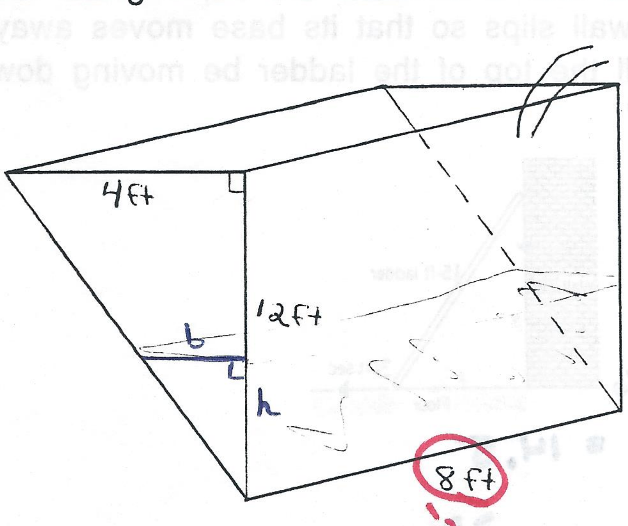
-3 no θ

$\frac{d\theta}{dt} = \frac{1}{60}(8)\left(\frac{12}{13}\right)^2$

$\frac{d\theta}{dt} = \cancel{1.48}$ .114 rad/sec

4. A trough is 8 feet long, 4 feet across the top, and 12 feet high (see figure). If water is being pumped into the trough at 1.5 cubic feet per minute, how fast is the water level rising when the water is 2 feet deep?

1.5 ft³/min $= \frac{dV}{dt}$ $\frac{dh}{dt} = ?$ $h = 2$



$V = Bh$

$V = \left(\frac{1}{2}bh\right)\cdot 8$

$V = \frac{1}{2}\left(\frac{h}{3}\right)h\cdot 8$

$V = \frac{4}{3}h^2$

$\frac{dV}{dt} = \frac{8}{3}h\frac{dh}{dt}$

$1.5 = \frac{8}{3}(2)\frac{dh}{dt}$

$\frac{dh}{dt} = .28125$

$\frac{b}{4} = \frac{h}{12}$

-3 wrong $\rightarrow b = \frac{h}{3}$

Name: 40 pts

Calculus - Related Rates Test 2012

Solve each related rate problem. Show your equations and a diagram if needed. (10 pts ea)

1. Hannah the vegetarian is looking at her tofu/veggie pizza and notices it's growing at a rate of 12 mm per second! What is the rate of change in the ginormous pizza's area when the diameter is 42 mm?

$\frac{dr}{dt} = 12$ $\frac{dA}{dt} = ?$ $r = 21$

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi (21)(12)$

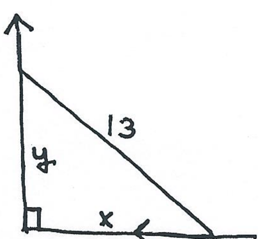
−9

$= \cancel{3166.7}\ mm^2/s$

$= 1583.36$

2. Karlie is moving in with Brandon and her bed won't fit through the door so she has to pull it up the outside of the building. Her bed is 13 feet long!!! The end of the bed is sliding along the ground at 5 m/sec. How fast is the end of the bed moving up the wall when the bed is 5 feet from the wall?

$x = 5$ $? = \frac{dy}{dt}$



$\frac{dx}{dt} = -5\ m/s$

$x^2 + y^2 = 13^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$5(-5) + 12\frac{dy}{dt} = 0$

$5^2 + y^2 = 13^2$

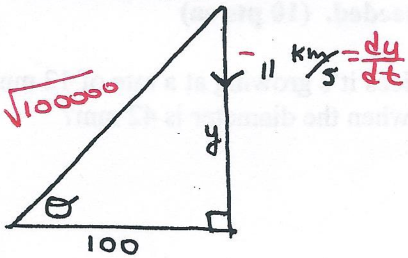
$y = 12$

$\frac{dy}{dt} = \frac{25}{12}$

$= 2.08\ ft/s$

−1

3. A meteor is speeding towards earth at 11 km/s. Astronomers are tracking the meteor at a point 100 km away from the estimated point of impact on earth. At what rate is the angle of elevation of the telescope toward the meteor changing when the meteor is 300 km above the estimated point of impact?



$\frac{d\theta}{dt} = ?\quad y = 300$

$\sec\theta = \frac{\sqrt{100000}}{100}$

$\tan\theta = \frac{y}{100}$

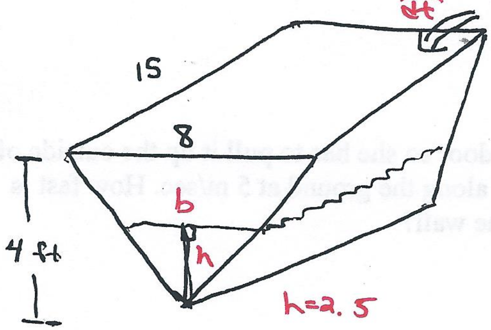
$\sec^2\theta\,\frac{d\theta}{dt} = \frac{1}{100}\frac{dy}{dt}$

$\frac{100000}{10000}\frac{d\theta}{dt} = \frac{1}{100}(-11)$

$\frac{d\theta}{dt} = \frac{-11}{1000} = -.011\ \frac{rad}{s}$

4. Katt Melly wishes to have his hot tub filled with hot nacho cheese at 0.5 cubic feet per minute. His hot tub is 4 feet deep and in the shape of a triangular prism. It is 15 feet long and 8 feet across. Find the rate of change of the depth of the hot nacho cheese when it is 2.5 feet deep.

$\frac{dv}{dt} = 0.5\ ft^3/min$



$h = 2.5$

$\frac{b}{8} = \frac{h}{4}$

$b = 2h$

$V = \frac{1}{2}bh \cdot 15$

$V = 15h^2$

$\frac{dv}{dt} = 30h\frac{dh}{dt}$

$.5 = 30(2.5)\frac{dh}{dt}$

$\frac{dh}{dt} = .0067\ ft/min$