Basic Limit Rules: Plugin, Factor/Cnx, Do Math, Infinite, Rationalization, Graphs, One sided, Trig

$\lim_{x \to \pm\infty} \frac{c}{x^n} = 0, n > 0$ $\lim_{x \to +\infty} \ln x = +\infty$ $\lim_{x \to 0^+} \ln x = -\infty$ $\lim_{x \to +\infty} e^x = +\infty$ $\lim_{x \to -\infty} e^x = 0$ $\lim_{x \to +\infty} e^{-x} = 0$

$\lim_{x \to -\infty} e^{-x} = +\infty$ $\lim_{x \to 0} \frac{\sin x}{x} = 1$ $\lim_{x \to 0} \frac{\tan x}{x} = 1$ $\lim_{x \to 0} \frac{1-\cos x}{x} = 0$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Basic Derivative Rules (u = f(x), v = g(x), c = any real number)

Power Rule: $(cx^n)' = cnx^{n-1}$

Product Rule: $(uv)' = u'v + v'u$

Quotient Rule: $(\frac{u}{v})' = \frac{u'v - v'u}{v^2}$

Chain Rule: $(cu^n)' = cnu^{n-1}u'$

Implicit: $x'ydx + y'xdy$ solve for dy/dx

$(u \pm v)' = u' \pm v'$

$|u|' = u'\frac{u}{|u|}$

$(\sin u)' = (\cos u)u'$

$(\cos u)' = (-\sin u)u'$

$(\tan u)' = (\sec^2 u)u'$

$(\cot u)' = (-\csc^2 u)u'$

$(\sec u)' = (\sec u \tan u)u'$

$(\csc u)' = (-\csc u \cot u)u'$

$(\ln u)' = \frac{u'}{u}$

$(\log_a u)' = \frac{u'}{u}\frac{1}{\ln a}$

$(e^u)' = u'e^u$

$(a^u)' = u'a^u \ln a$

$(\sin^{-1} u)'$ *or* $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$

$(\tan^{-1} u)'$ *or* $(\arctan u)' = \frac{u'}{1+u^2}$

$(\sec^{-1} u)'$ *or* $(\operatorname{arcsec} u)' = \frac{u'}{|u|\sqrt{u^2-1}}$

$(\cos^{-1} u)'$ *or* $(\arccos u)' = \frac{-u'}{\sqrt{1-u^2}}$

$(\cot^{-1} u)'$ *or* $(\operatorname{arccot} u)' = \frac{-u'}{1+u^2}$

$(\csc^{-1} u)'$ *or* $(\operatorname{arccsc} u)' = \frac{-u'}{|u|\sqrt{u^2-1}}$

Basic Integration Formulas (u = f(x), C or a = any real number)

$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$

$\int \sin u \, du = -\cos u + C$

$\int \cos u \, du = \sin u + C$

$\int \sec^2 u \, du = \tan u + C$

$\int \csc^2 u \, du = -\cot u + C$

$\int \sec u \tan u \, du = \sec u + C$

$\int \csc u \cot u \, du = -\csc u + C$

$\int \frac{du}{u}$ *or* $\int u^{-1} du = \ln|u| + C$

$\int \tan u \, du = -\ln|\cos u| + C$

$\int \cot u \, du = \ln|\sin u| + C$

$\int \sec u \, du = \ln|\sec u + \tan u| + C$

$\int \csc u \, du = -\ln|\csc u + \cot u| + C$

$\int e^u du = e^u + C$

$\int a^u \, du = \frac{1}{\ln a} a^u + C$

$\int_a^b f(x)dx = F(a) - F(b)$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}(\frac{|u|}{a}) + C$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$

$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}(\frac{u}{a}) + C$

$y = \log_b x \Leftrightarrow b^y = x$

$y = \ln x \Leftrightarrow e^y = x$

$\ln x = \log_e x$

$\ln e = 1, \log 10 = 1$

$\ln 1 = 0, \log 1 = 0$

$\log x^r = r \log x$

$\log(xy) = \log x + \log y$

$\log(\frac{x}{y}) = \log x - \log y$

$\log_b a = \frac{\log a}{\log b}$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{-A}{B}$ $m_T = \frac{-1}{m}$

Line: $y - y_1 = m(x - x_1)$ or Ax+By=C

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

16. **Integration by Parts:**

$\int u\,dv = uv - \int v\,du$

Type 1: $\int x^n e^{ax}dx,\ \int x^n \sin ax\,dx,\ \int x^n \cos ax\,dx$, Let $u = x^n$ and $dv = e^{ax}dx$, $\sin ax\,dx$, $\cos ax\,dx$

Type 2: $\int x^n \ln x\,dx, \int x^n \sin^{-1}ax\,dx,\ \int x^n \tan^{-1}ax\,dx$, Let $u = \ln x, \sin^{-1}ax, \tan^{-1}ax$ and $dv = x^n\,dx$

Type 3: $\int e^{ax}\sin bx\,dx,\ \int e^{ax}\cos bx\,dx$ Let $u = \sin bx, \cos bx$ and $dv = e^{ax}\,dx$

17. **Powers of Sine and Cosine:**

1. $\int \sin^{odd}x\cos^n x\,dx$: Save 1 $\sin x$ for du and convert other $\sin^m x$ to $\cos x$ using $\sin^2 x = 1-\cos^2 x$
2. $\int \sin^n x\cos^{odd}x\,dx$: Save 1 $\cos x$ for du and convert other $\cos^m x$ to $\sin x$ using $\cos^2 x = 1-\sin^2 x$
3. $\int \sin^{even}x\cos^{even}x\,dx$: use $\sin^2 x = \dfrac{1-\cos 2x}{2}$ and $\cos^2 x = \dfrac{1+\cos 2x}{2}$ to convert to $\cos^{odd}2x$ and use above rules

18. **Powers of Tangent and Secant:**

1. $\int \sec^{even}x\tan^n x\,dx$: Save $\sec^2 x$ for du and convert other $\sec^m x$ to $\tan x$ using $\sec^2 x = 1+\tan^2 x$
2. $\int \sec^n x\tan^{odd}x\,dx$: Save $\sec x\tan x$ for du and convert other $\tan^m x$ to $\sec x$ using $\tan^2 x = \sec^2 x - 1$
3. $\int \tan^{even}x\,dx$: Convert a $\tan^2 x$ to $\sec^2 x - 1$ and expand
4. $\int \sec^{odd}x\,dx$: Use intergration by parts

Special Summation Formulas

19. $\sum_{i=1}^{n} c = cn \qquad \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \dfrac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \left(\dfrac{n(n+1)}{2}\right)^2 \qquad \text{Area} = \lim_{n\to\infty}\sum_{i=1}^{n}\left(\dfrac{b-a}{n}\right)f(a+bi)$

width height

Areas, Volumes, Arc Length and Surface Areas

20. **AREA BETWEEN:** $A = \int_a^b [f(x)] - [g(x)]\,dx$; $f(x)$ is upper, $g(x)$ is lower; $f(y)$ is right, $g(y)$ is left

21. **DISK:** $V = \pi\int_a^b [f(x)]^2 dx$; (horizontal axis) $f(x)$ is radius or $V = \pi\int_a^b [f(y)]^2 dy$; (vertical axis) $f(y)$ is radius

22. **WASHER:** $V = \pi\int_a^b [f(x)]^2 - [g(x)]^2 dx$; $f(x)$ is outer radius and $g(x)$ is inner radius

23. **SHELL:** $V = 2\pi\int_a^b r(x)[f(x)-g(x)]\,dx$; $r(x)$ is radius, $f(x)$ is upper and $g(x)$ is lower

24. **ARC LENGTH:** $s = \int_a^b \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$; $s = \int_a^b \sqrt{1+\left(\dfrac{dx}{dy}\right)^2}\,dy$

25. **SURFACE AREA:** $S = 2\pi\int_a^b r(x)\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$; $r(x)$ is radius $S = 2\pi\int_a^b r(y)\sqrt{1+\left(\dfrac{dx}{dy}\right)^2}\,dy$; $r(y)$ is radius